Homework 3

Problem 1: Year-on-year quarterly data and ARMA dynamics

A substantial amount of quantity data, such as earnings, exhibit seasonalities. These

can be hard to model. It is therefore common to use so-called Year-on-Year data (e.g., Q1

earnings vs Q1 earnings a year ago, Q2 earnings vs Q2 earnings a year ago, etc). In this

problem we will see that such a practice can induce MA-terms due to the overlap in the

quarterly year-on-year observations.

Assume the true quarterly log market earnings follow:

where and is i.i.d. over time t.

The earnings data you are given is year-on-year earnings growth, which in logs is:

1. Assume . Derive autocovariances of order 0 through 5 for . I.e., for .

Based on where we can get and :

1. Assume . Determine the number of AR lags and MA lags you need in the ARMA(p,q) process for . Give the associated AR and MA coefficients.

The red rectangle shows the overlapping between and . Thus, this demonstrates that has a MA coefficient of ().

Since only contains terms, which are all white noise, we can also say that has a AR coefficient of ().

Therefore, has an process.

Problem 2: Market-timing and Sharpe ratios

Much of this class is about prediction. In this problem you will derive how market

timing can improve the unconditional Sharpe ratio of a fund. The market timing is based

on "forecasting regressions" akin to those we undertake in a VAR. However, we are only

forecasting one period ahead here.

Assume you have an estimate of expected annual excess market returns for each time t, called . You estimate the regression

and obtain   and . Further, the sample mean and standard deviation of are both 5%.

1. Calculate the standard deviation of excess returns based on the information given.
2. Calculate the R2 of the regression based on the information given.
3. Calculate the sample Sharpe ratio of excess market returns based on the information given.
4. Recall from investments that myopic investors chooses a fraction of wealth

in the risky asset (the market) at each time , where we assume risk aversion coefficient, , equals 40/9. Further, assume that the residuals are i.i.d., so for all . Given this, calculate the weight the investor chooses to hold in the risky asset if and if . What is conditional Sharpe ratio in each of these cases?

1. R Assume T is large (i.e., T→1) and that is either 0% or 10% at each time t, with equal probability (0:5).
2. What is the unconditional average excess return for an investor that holds t each period?
3. What is the unconditional standard deviation? The following may be helpful for calculating the unconditional variance. You could also simulate a very long series 2 to check your math.